Technical Note 1971-33

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Statistics of the Spectral Ratio and Log-Likelihood Ratio Seismic Discriminants

24 September 1971

Prepared for the Advanced Research Projects Agency under Electronic Systems Division Contract F19628-70-C-0230 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

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LINCOLN LABORATORY

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The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. This research is a part of Project Vela Uniform, which is sponsored by the Advanced Research Projects Agency of the Department of Defense under Air Force Contract F19628-70-C-0230 (ARPA Order 512).

ABSTRACT

The effect of noise variations on two of the short period seismic discriminants, spectral ratio and log-likelihood ratio is studied. The mean square errors of two different estimators of these two discriminants are evaluated and some theoretical and experimental results for two seismic signals in seismic noise are presented.

Accepted for the Air Force
Joseph R. Waterman, Lt. Col., USAF
Chief, Lincoln Laboratory Project Office

I. INTRODUCTION

During the last ten years, a considerable amount of effort has been directed toward the study of natural and manmade seismic sources and their differentiating features. Based on theoretical and experimental investigations, a number of discriminating features of earthquakes and underground nuclear explosions have been discovered, and their identification capabilities, on the basis of data from large magnitude events, have been demonstrated[1,2]. When the event magnitude diminishes, the level of the background noise at receiver becomes significant and causes random variations in the seismic discriminants. Thus a discriminating feature which may be capable of differentiating between large magnitude seismic events, becomes sensitive to noise variations at small signal-to-noise ratios and its identification capability declines drastically. Therefore, the identification capability of each seismic discriminant must be studied as a function of signal-to-noise ratio. This study requires the evaluation of the statistics of each seismic discriminant.

In this report, we will consider two of the short period seismic discriminants, spectral ratio and log-likelihood ratio, treat them as random variables and study their statistics. By statistics of a random variable, in this study, we mean the mean and mean square error of that variable.

II. DEFINITIONS OF SPECTRAL RATIO AND LOG-LIKELIHOOD RATIO

Spectral ratio seismic discriminant is defined as the ratio of energy in the frequency band 1.45-1.95 Hz to that in the frequency band 0.35-0.85 Hz of the P-wave energy spectrum i.e.,

$$SR \equiv \frac{Z_H}{Z_L} \tag{1}$$

where Z_L and Z_H are the energy in the low and high frequency bands. This definition differs from the one given by Lacoss[2] in that he uses the voltage spectrum instead of energy spectrum. The primary reason for the use of energy spectrum is that it is mathematically more convenient to study the statistics of the energy spectrum estimates than those of the voltage spectrum estimates. In the absence of noise, the situation which is approximated by very large events, Z_L and Z_H depend on the event magnitude, source mechanism, transmission path and etc. Since SR is a ratio, we can consider normalized Z_L and Z_H. To compute normalized Z_L and Z_H for a large magnitude event, we normalize that event by its maximum amplitude on the unfiltered trace and then compute its energy in the low and high frequency bands. Figs. 1 and 2 show scatter diagrams of $\log_{10} Z_L$ and $\log_{10} Z_H$ for all of the shallow earthquakes and presumed explosions of $m_b \geq 5.0$ used by Lacoss[2]. The straight lines in each figure are the least square fits for the earthquake and presumed explosion populations. If we define

$$R_L \equiv \log_{10} Z_L \tag{2}$$

and

$$R_H \equiv \log_{10} Z_H \tag{3}$$

and assume that at a given magnitude R_L and R_H are random variables, then the mean and standard deviation of each of these variables, for a given population, are obtained by the height of the line corresponding to that population at the given magnitude and the population scatter about that line, respectively.

The log-likelihood ratio of these two variable is defined by

$$\gamma \equiv \ln \frac{p(R_L, R_H: \text{ given presumed explosions})}{p(R_L, R_H: \text{ given earthquakes})} \tag{4}$$

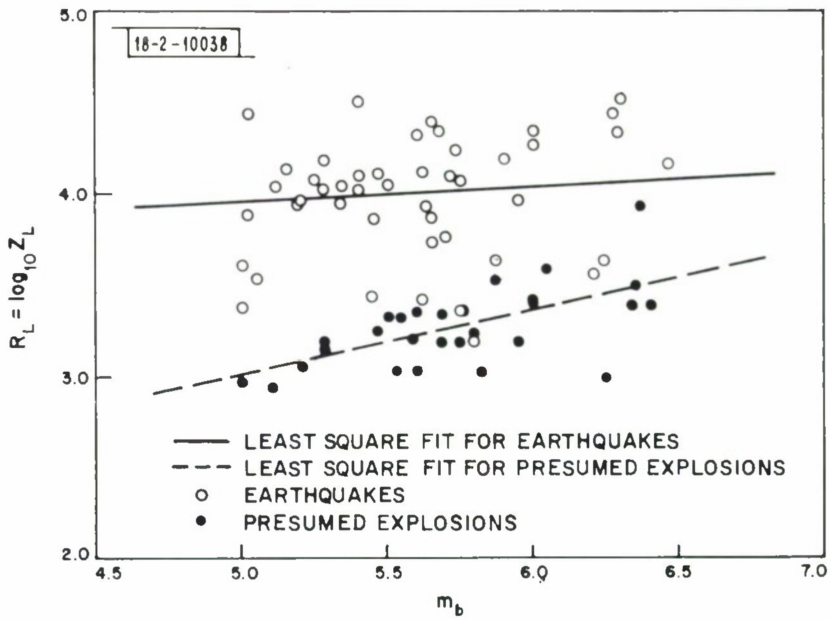

Fig. 1. Scatter diagram of the energy in the low frequency band for 42 earthquakes and 28 presumed explosions. Each event has been normalized to have a unity maximum amplitude.

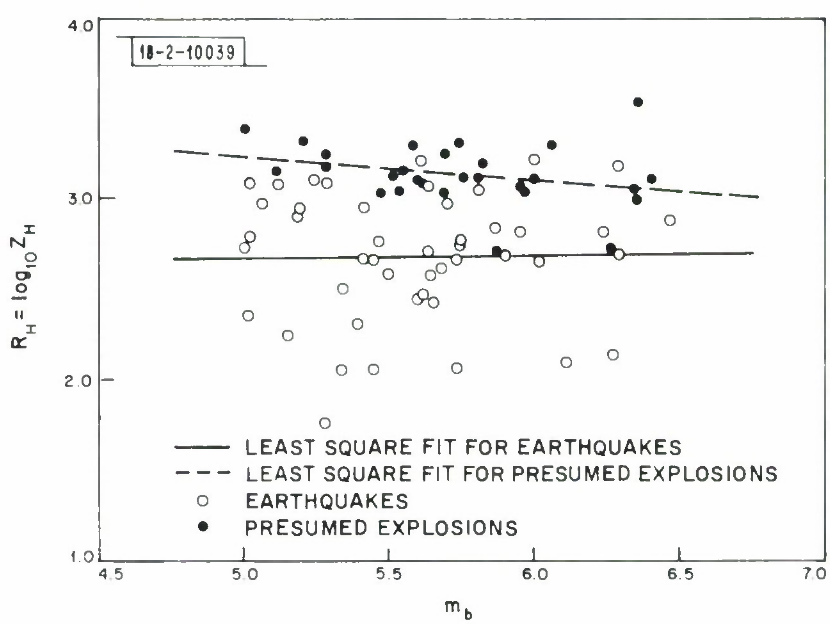

Fig. 2. Scatter diagram of the energy in the high frequency band for 42 earthquakes and 28 presumed explosions. Each event has been normalized to have a unity maximum amplitude.

where $p(R_L, R_H$: given a population) denotes the joint density of R_L and R_H for that population. If we assume that R_L and R_H are independent normally distributed random variables, then

$$p(R_L, R_H: \text{ given presumed explosions}) =$$

$$\frac{1}{2\pi\sigma_{XL}\sigma_{XH}} \exp\left[-\frac{(R_L - m_{XL})^2}{2\sigma^2_{XL}} - \frac{(R_H - m_{XH})^2}{2\sigma^2_{XH}}\right] \tag{5}$$

where m_{XL}, m_{XH}, σ_{XH} and σ_{XL} denote the means and standard deviations of R_L and R_H for the presumed explosions' population. The joint probability density of R_L and R_H, given earthquakes, is obtained by replacing X by Q in Eq. (5). Substituting Eq. (5) and its dual for the earthquakes into Eq. (4) and expanding it, we obtain

$$\gamma = \frac{(R_L - m_{QL})^2}{2\sigma^2_{QL}} - \frac{(R_L - m_{XL})^2}{2\sigma^2_{XL}} + \frac{(R_H - m_{QH})^2}{2\sigma^2_{QH}}$$

$$- \frac{(R_H - m_{XH})^2}{2\sigma^2_{XH}} + \ln\frac{\sigma_{QL}\sigma_{QH}}{\sigma_{XL}\sigma_{XH}} \quad . \tag{6}$$

We now observe that if $\sigma_{XL} = \sigma_{QL} = \sigma_L$, $\sigma_{XH} = \sigma_{XH} = \sigma_{QH} = \sigma_H$ and $\frac{m_{XL} - m_{QH}}{\sigma^2_H} = \frac{m_{QL} - m_{XL}}{\sigma_L^2} = 1$, then γ reduces to

$$R_H - R_L + \text{constant}$$

which is the same as the log of the spectral ratio except for a constant. This result implies that the spectral ratio is a simplified version of the log-likelihood ratio discriminant. Kelly[3] has applied the log-likelihood ratio as a seismic discriminant and he has concluded that this discriminant performs the same as or better than the spectral ratio.

In order to classify a new large magnitude event using either of these two discriminants, we must measure m_b, Z_L and Z_H, and evaluate the corresponding discriminant and compare it with a threshold. Note that the threshold for the spectral ratio as well as m_{XL}, m_{XH}, m_{QL} and m_{QH} for the log-likelihood ratio are functions of m_b.

Thus far we have assumed that the noise contributions to Z_L and Z_H are small by considering large magnitude events. When the relative noise level becomes significant we must replace Z_L and Z_H by their estimates and use the estimates of SR and γ for classification purposes. Estimates of SR and γ, for a seismic event observed in noise, are random variables whose statistics vary with signal-to-noise ratio. Evaluation of the statistics of these two discriminants is the topic of the next section.

III. MEAN AND MEAN SQUARE ERROR OF SR AND γ ESTIMATES

In order to evaluate the mean and mean square error of $\widehat{SR}$ and $\hat{\gamma}$, (estimate of SR and γ) we make the simplifying assumption that the Fourier coefficients of the observed time series,

$$x_i = s_i + n_i \; ; \; i = 1,2,\ldots,M \tag{7}$$

are statistically independent complex random variables. This assumption is not unreasonable because, according to the Spectral Decomposition Theorem, when the time window becomes large the Fourier coefficients, X_i's, become independent of one another. Under this assumption $\hat{Z}_L$ and $\hat{Z}_H$ (estimates of Z_L and Z_H), which consist of the sum of $\widehat{|S_i|^2}$ (estimate of $|S_i|^2$) in the low and high frequency bands, become independent. Consequently, the joint probability density of Z_L and Z_H, which is required for the evaluation of the statistice $\widehat{SR}$ and $\hat{\gamma}$ reduces to the product of their individual

densities. In other words, the problem of evaluating the mean and mean square error of $\widehat{SR}$ and $\hat{\gamma}$ reduces to the problem of evaluating the probability density of an energy estimate in a frequency band. A detailed discussion of this problem and evaluation of the mean and mean square error of any estimate of SR and γ is given in the author's doctoral dissertation[4].

In the next section, we will present some theoretical and experimental results for the following two energy estimators (defined in Reference 4):

(i)

$$\text{Conventional energy estimator} \equiv \Sigma \left|X_i\right|^2$$

and (ii)

$$\text{ALPHA-C energy estimator} \equiv \Sigma\left[\,\left|X_i\right| - c\sigma_i \exp\left(-\frac{\alpha\left|X_i\right|^2}{\sigma_i^2}\right)\right]^2$$

where c and α are arbitrary constants and $2\sigma_i^2$ is the noise power at the i'th frequency component. Note that ALPHA-C estimator is a function of the observed value as well as the noise statistics. For the evaluation of numerical results in this report we have chosen $c = 1.45$ and $\alpha = 0.1$.

IV. NUMERICAL RESULTS

For the numerical evaluation of the mean and mean square error of $\widehat{SR}$ and $\hat{\gamma}$ as function of SNR, we need the following quantities:

(i) the noise power density spectrum which is chosen to be the estimate of the power density spectrum of 500 seconds of a seismic noise process observed at a short period seismometer,

(ii) the signal energy density spectrum which is chosen to be the energy spectrum of 10 seconds of a large magnitude seismic signal recorded by a short period instrument,

and

(iii) the frequency bands of interest, the frequency increment, and hence the number of frequency components in each frequency band.

The short period seismic data is sampled every 0.05 seconds and a frequency increment of 10/1024 Hz is chosen. The low and high frequency bands of interest are 0.35-0.85 Hz and 1.45-1.95 Hz, respectively. From these the number of Fourier coefficients in each frequency band is obtained.

Fig. 3 shows the noise power density spectrum as well as the energy density spectra of LASA short period beams of a large magnitude presumed explosion (signal no. 1) and a large magnitude earthquake (signal no. 2). These spectra are normalized to have unity energy in the frequency band of 0.0-2.0 Hz. The parameters associated with these two signals are as follows:

(i) for signal no. 1 (presumed explosion)

$m_b = 5.75$	$SR = 0.732$	$\gamma = 5.2$	
$m_{XL} = 3.28$	$m_{QL} = 4.01$	$m_{XH} = 3.13$	$m_{QH} = 2.68$
$\sigma^2_{XL} = 0.027$	$\sigma^2_{QL} = 0.1$	$\sigma^2_{XH} = 0.027$	$\sigma^2_{QH} = 0.123$

and (ii) for signal no. 2 (earthquake)

$m_b = 6.0$	$SR = 0.0246$	$\gamma = -16.5$	
$m_{XL} = 3.37$	$m_{QL} = 4.03$	$m_{XH} = 3.1$	$m_{QH} = 2.68$
$\sigma^2_{XL} = 0.027$	$\sigma^2_{QL} = 0.1$	$\sigma^2_{XH} = 0.027$	$\sigma^2_{QH} = 0.123$

We now present the mean square errors of $\hat{Z}_L$ and $\hat{Z}_H$, $\widehat{SR}$ and $\hat{\gamma}$ as functions of SNR. The experimental results are obtained by adding the same signal into 50 noise segments, each of 10 seconds duration. In order to control the signal-to-noise ratio, we keep the noise level constant and vary the signal amplitude.

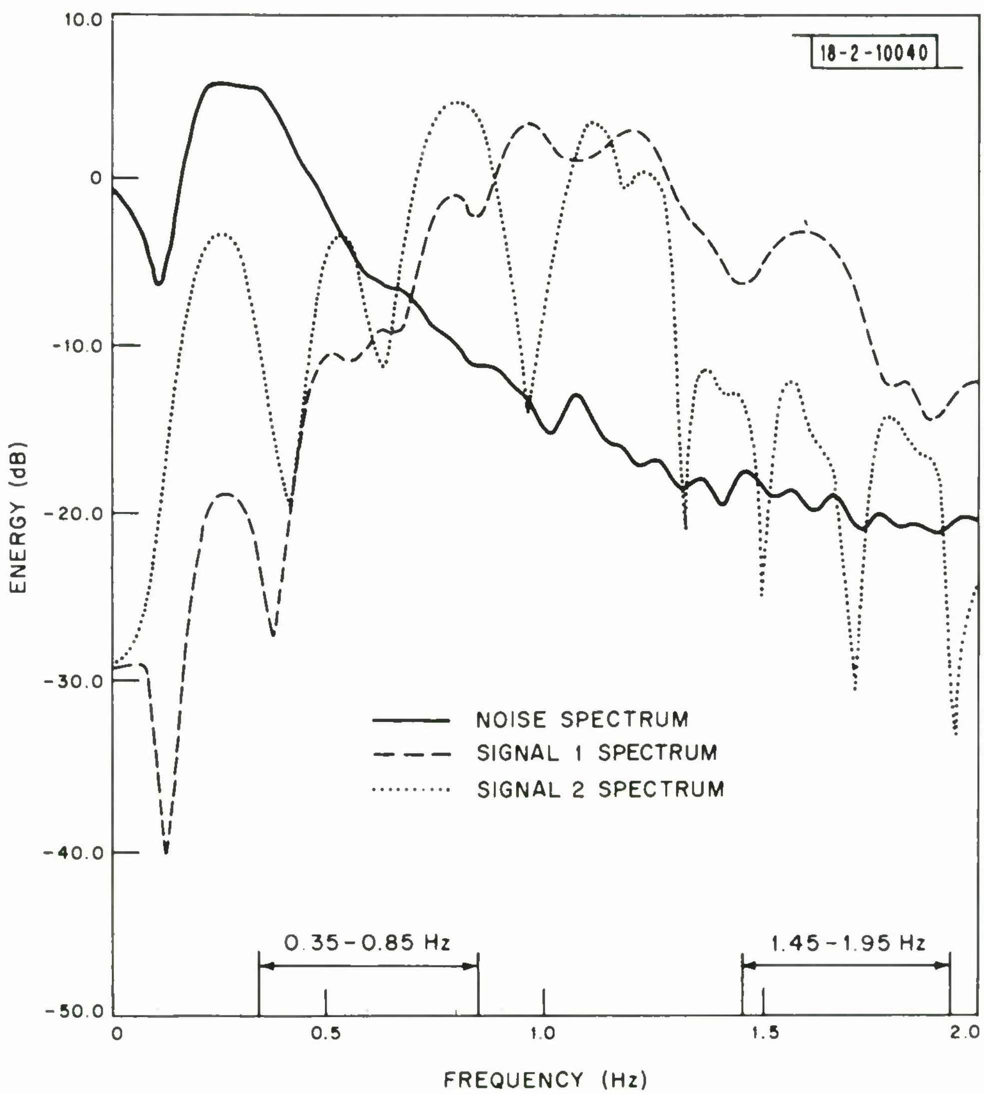

Fig. 3. Power density spectrum of 500 seconds of a seismic noise process and energy density spectra of two seismic signals recorded by a short period vertical instrument. Signal no. 1 corresponds to a presumed explosion and signal no. 2 to an earthquake. The total energy in each of these spectra is unity.

In the process of evaluating the mean square error of $\widehat{SR}$ and $\hat{\gamma}$ at each SNR, we obtain the probability density of the energy estimates in the two frequency bands. Fig. 4 shows the cumulative probability distribution, $P(\hat{Z})$, of the conventional and ALPHA-C estimates of energy in the low frequency band for signal no. 1. For a given value of the variable $\hat{Z}$, say Z_a, $P(Z_a)$ gives the probability that $\hat{Z} \leq Z_a$. The actual signal energy in this frequency band is 1.58 and the signal energy to noise energy (SNR) in this band is -2.1 db. In this figure are also shown the histograms of the conventional and ALPHA-C energy estimates obtained from experimental results. The discrepancy between the theoretical and experimental results is due to two factors (i) dependence between the Fourier coefficients in the frequency band of interest and (ii) the difference between the sample moments and statistical moments of a random variable.

Another by-product of this investigation is the evaluation of the mean and variance of energy estimates in a frequency band. Figs. 5 and 6 show the normalized (by the actual value of energy) root mean square error of the conventional and ALPHA-C energy estimates in the low and high frequency bands for signals no. 1 and 2. These figures present the statistical (theoretical) as well as the arithmetic (experimental) root mean square errors as functions of the signal multiplier which is closely related to the SNR. Based on these results we conclude that the performance of the ALPHA-C energy estimators is superior to that of the conventional estimator at all signal-to-noise ratios.

Figs. 7 and 8 represent the statistical and arithmetic root mean square error of the conventional and ALPHA-C spectral ratio estimators for signals no. 1 and 2. Based on the results of Figs. 5, 6, 7 and 8, we observe that although the performance

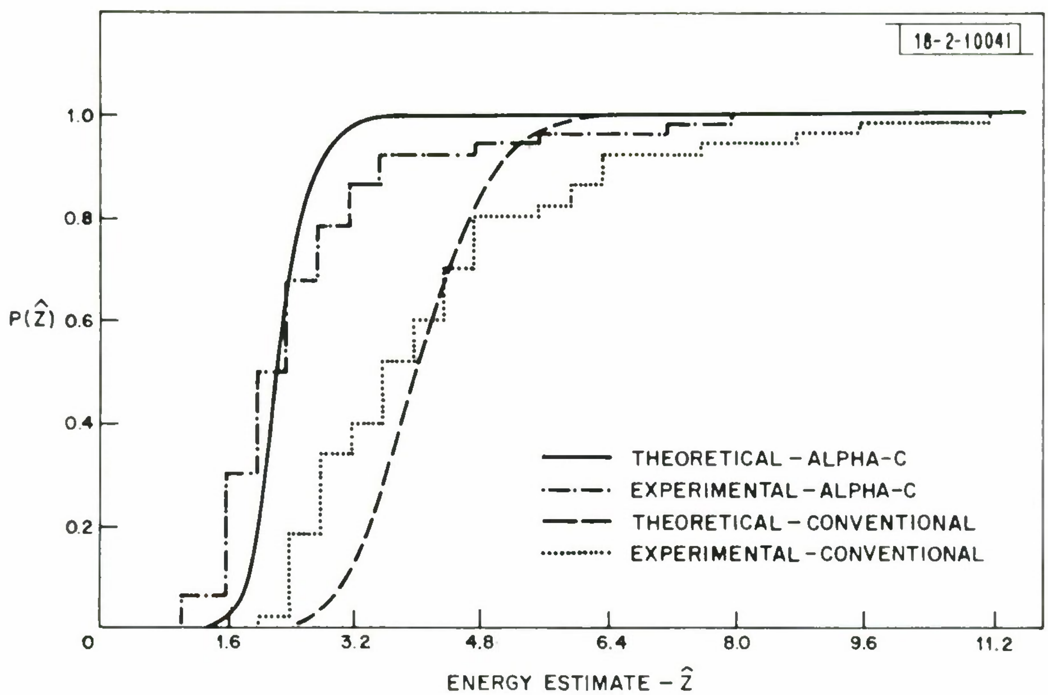

Fig. 4. Probability functions and histograms of the conventional and ALPHA-C estimators of energy in the low frequency band of signal no. 1 (presumed explosion) at a signal-to-noise ratio of -2.1 db.

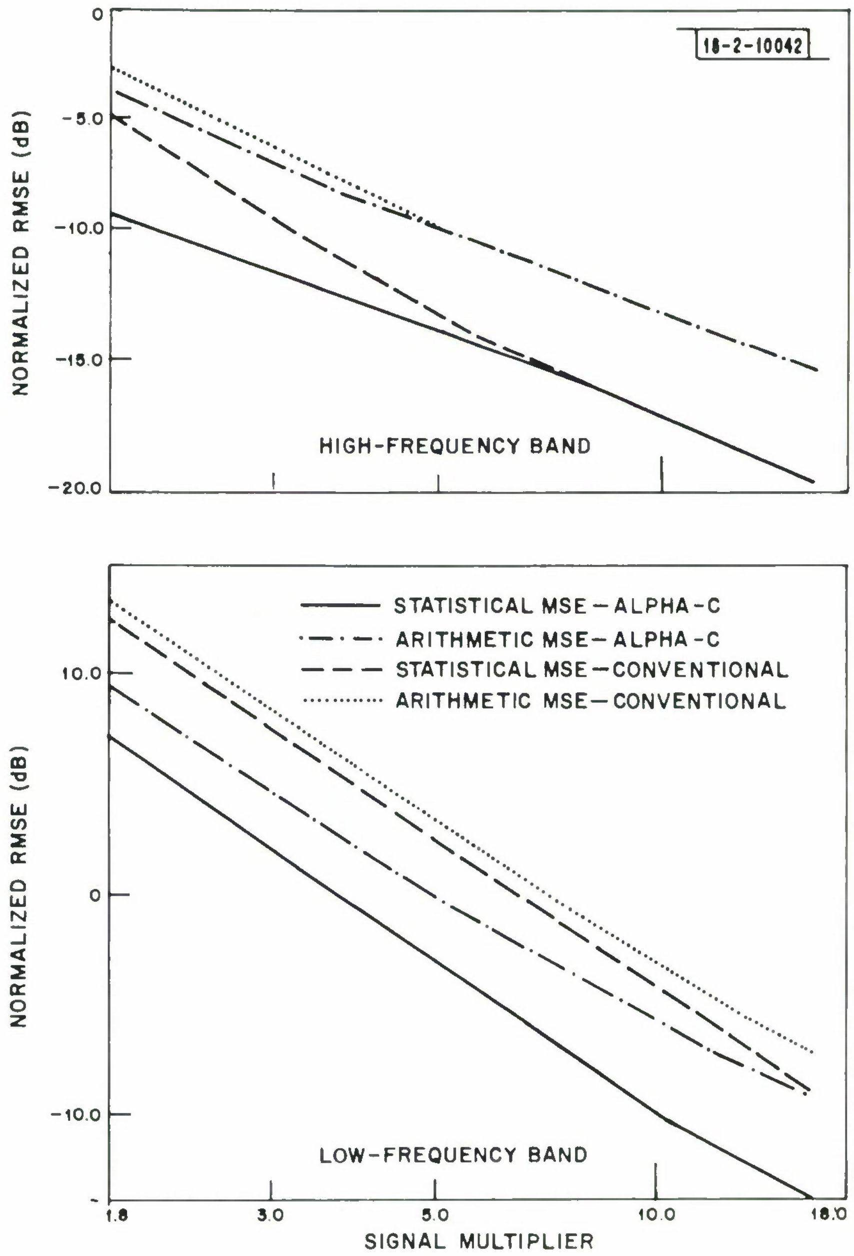

Fig. 5. Normalized root mean square error of the conventional and ALPHA-C estimators in the high and low frequency bands for signal no. 1 (presumed explosion).

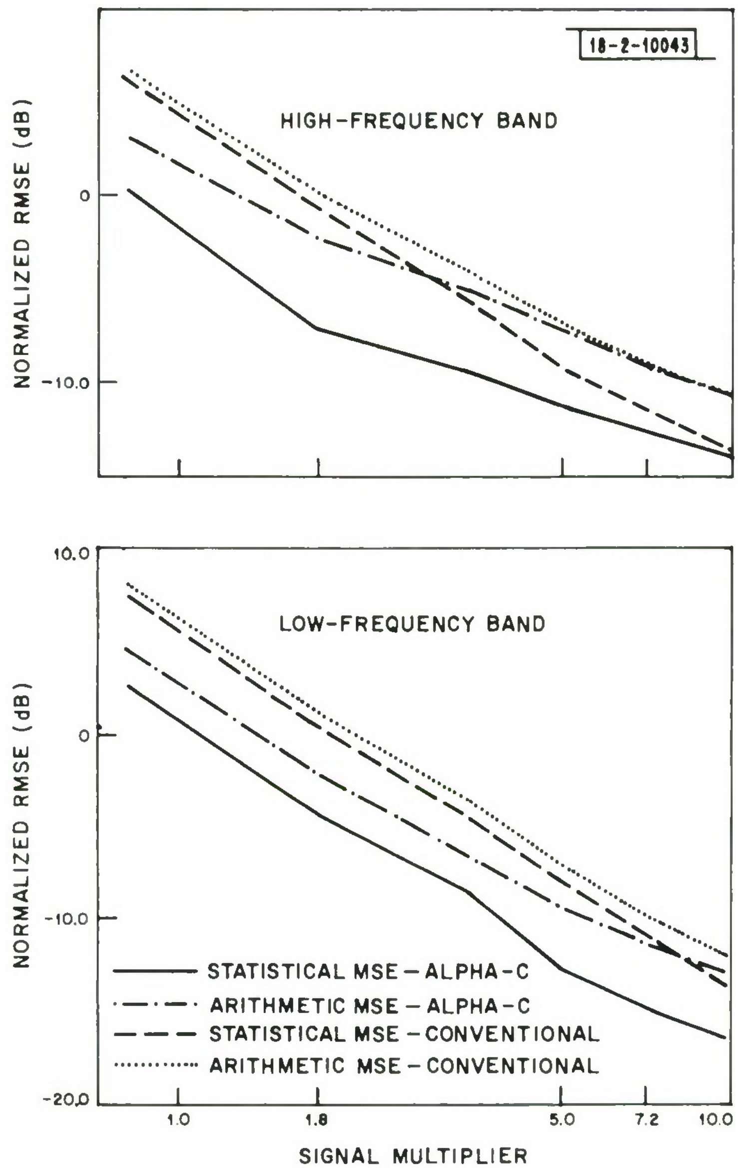

Fig. 6. Normalized root mean square error of the conventional and ALPHA-C energy estimators in the high and low frequency bands for signal no. 2 (earthquake).

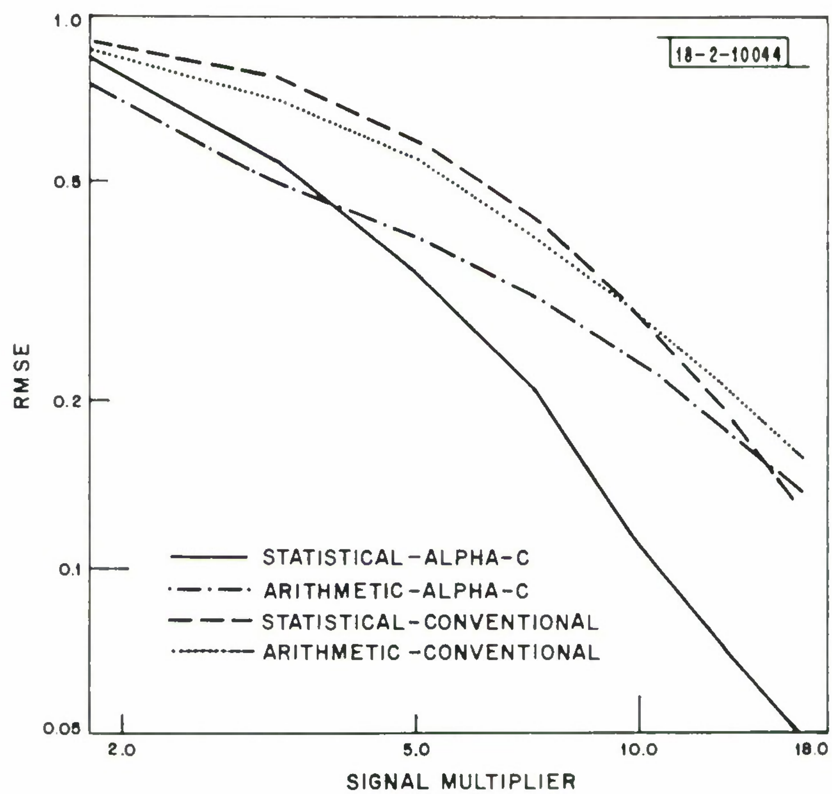

Fig. 7. Root mean square error of the conventional and ALPHA-C estimators of the spectral ratio for signal no. 1 (presumed explosion).

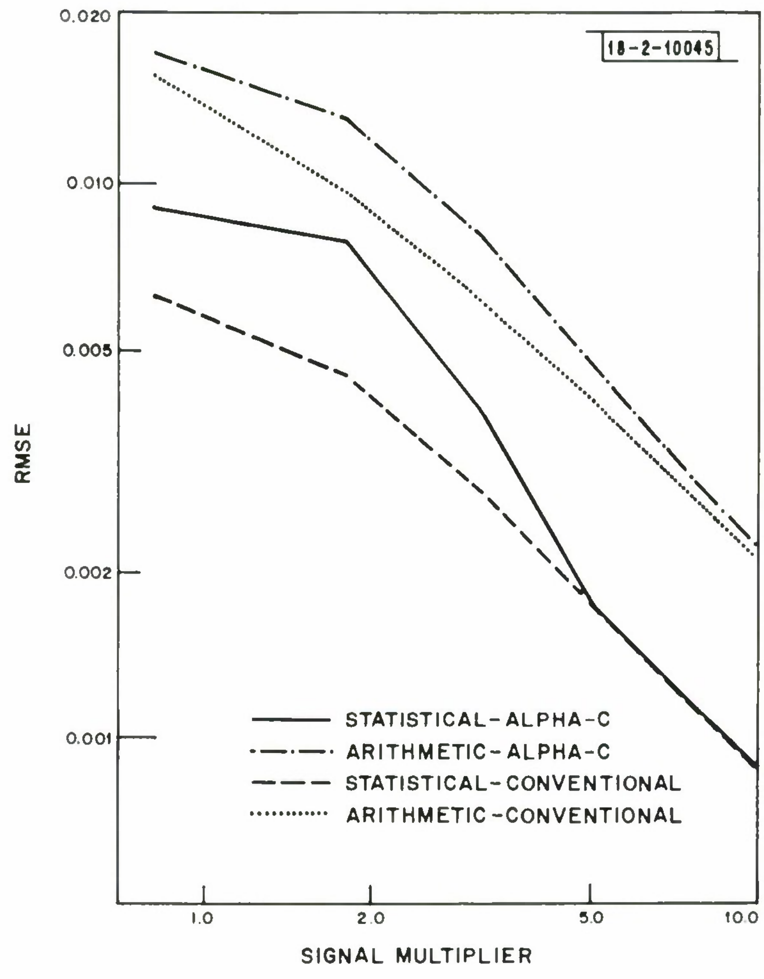

Fig. 8. Root mean square error of the conventional and ALPHA-C estimators of the spectral ratio for signal no. 2 (earthquake).

of the ALPHA-C energy estimator is superior to that of the conventional energy estimator, the ALPHA-C spectral ratio estimator does not perform as well as the conventional spectral ratio estimator in the case of signal no. 2. This is perhaps due to one of the properties of the ratio of two variables that may remain constant while both of the variables are changing. This is a poor quality of spectral ratio as a seismic discriminant.

Figs. 9 and 10 show the statistical and arithmetic root mean square error of the ALPHA-C and conventional estimators of the log-likelihood ratio for signals no. 1 and 2. These figures indicate that the performance of the ALPHA-C estimator of the log-likelihood ratio is superior to that of the conventional estimator of this discriminant for both signals.

Comparison of the mean square error of the spectral ratio estimates and the mean square error of the log-likelihood ratio estimates for signals no. 1, 2 and a few other signals indicates that the log-likelihood ratio discriminant reflects the variations of $\hat{Z}_L$ and $\hat{Z}_H$ in a more pronounced manner than the spectral ratio discriminant. Based on this property of the log-likelihood ratio and the fact that its identification capability, for large magnitude events, is the same as or better than the spectral ratio its application is highly recommended.

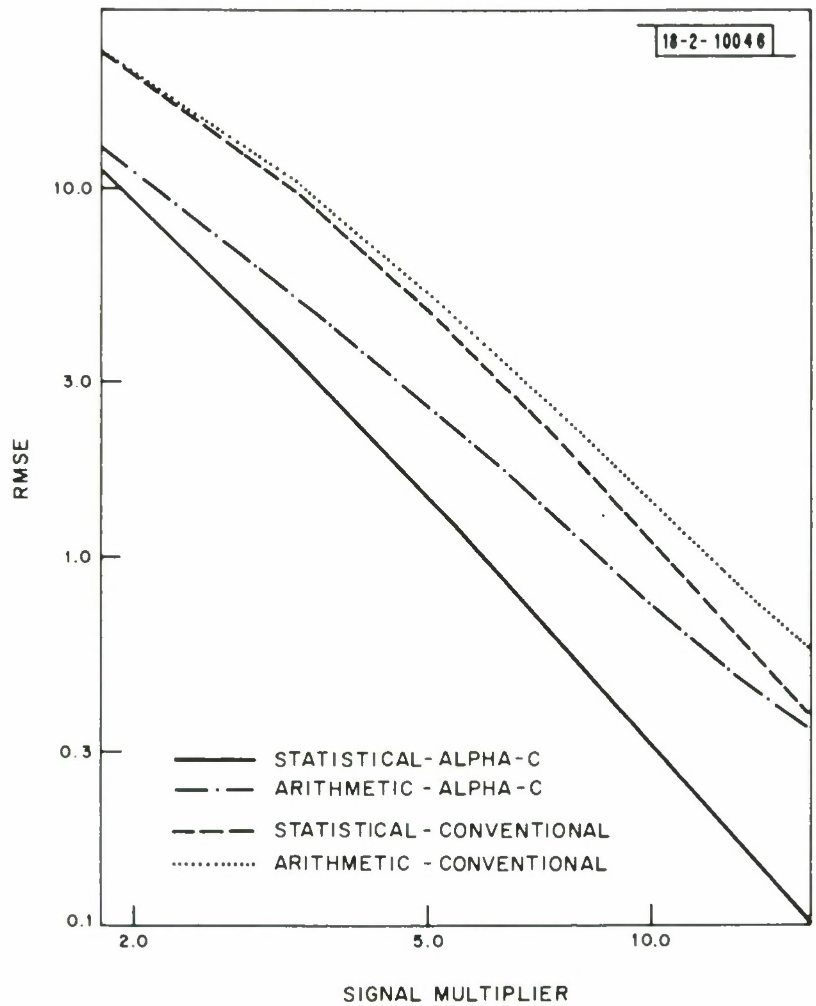

Fig. 9. Root mean square error of the conventional and ALPHA-C estimators of the log-likelihood ratio for signal no. 1 (presumed explosion).

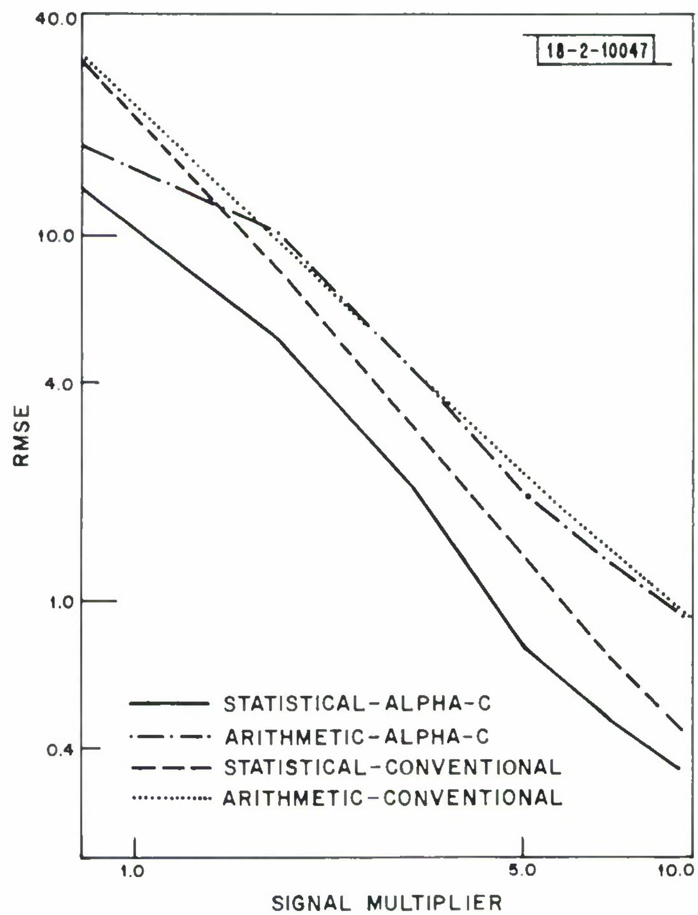

Fig. 10. Root mean square error of the conventional and ALPHA-C estimators of the log-likelihood ratio for signal no. 2 (earthquake).

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DOCUMENT CONTROL DATA – R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Lincoln Laboratory, M.I.T.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP None
3. REPORT TITLE Statistics of the Spectral Ratio and Log-Likelihood Ratio Seismic Discriminants	
4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)* Technical Note	
5. AUTHOR(S) *(Last name, first name, initial)* Mohajeri, Manouchehr	

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
24 September 1971	24	4

8a. CONTRACT OR GRANT NO. F19628-70-C-0230 b. PROJECT NO. ARPA Order 512 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Note 1971-33 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* ESD-TR-71-270
10. AVAILABILITY/LIMITATION NOTICES Approved for public release; distribution unlimited.	
11. SUPPLEMENTARY NOTES None	12. SPONSORING MILITARY ACTIVITY Advanced Research Projects Agency, Department of Defense

13. ABSTRACT

The effect of noise variations on two of the short period seismic discriminants, spectral ratio and log-likelihood ratio, is studied. The mean square errors of two different estimators of these two discriminants are evaluated and some theoretical and experimental results for two seismic signals in seismic noise are presented.

14. KEY WORDS

seismic discrimination	spectral ratio
noise variations	log-likelihood ratio

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